Limits and Continuity

Grade: «grade»

Subject: «subject»

Date: «date»

1 Answer?

A calculator may not be used on the following questions.

1. Evaluate the limit, if it exists: $\lim_{x \to 2} \frac{x^2 + x - 6}{2 - x}$.

(A) 5
(B) 3
(C) –3
(D) –5
(E) The limit does not exist.

$$\frac{(x+3)(x-2)}{-(x-2)}$$

2 Answer?

2. Evaluate the limit, if it exists: $\lim_{x \to 9} \frac{\sqrt{x-5}-2}{x-9}$

(A) $\frac{1}{4}$

(B) $-\frac{1}{4}$

(C) 1

(D) 0

(E) The limit does not exist.

$\left(\frac{\sqrt{x-5}-2}{x-9}\right)\left(\frac{\sqrt{x-5}+2}{\sqrt{x-5}+2}\right)$

$\frac{x-5-4}{(x-9)(\sqrt{x-5}+2)} = \frac{(x-9)}{(x-9)(\sqrt{x-5}+2)}$

3 Answer?

3. Evaluate the limit, if it exists: $\lim_{x\to 2} \dfrac{\frac{1}{x}-\frac{1}{2}}{x-2}$.

(A) $\frac{1}{4}$

(B) $-\frac{1}{4}$

(C) 1

(D) –1

(E) The limit does not exist.

$$\frac{\frac{2-x}{2x}}{x-2}$$

$$\frac{2-x}{2x} \cdot \frac{1}{x-2}$$

$$\frac{-(x-2)}{2x(x-2)}$$

4 Answer?

4. Evaluate the limit, if it exists: $\lim_{x\to 1} \frac{\tan^{-1} x}{\sin^{-1} x + 1}$.

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{\pi}{2}$

(E) $\frac{\pi}{2\pi + 4}$

$$\frac{\frac{\pi}{4}}{\frac{\pi}{2} + \frac{2}{2}}$$

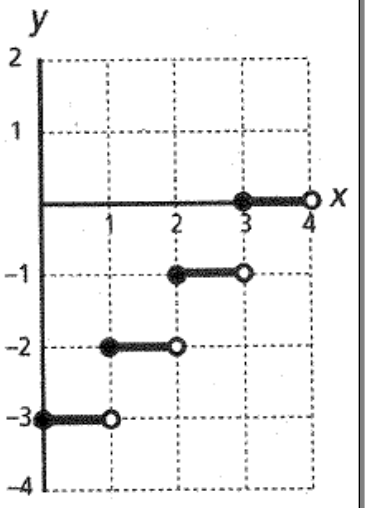
$$= \frac{\frac{\pi}{4}}{\frac{\pi+2}{2}} = \frac{\pi}{4} \cdot \frac{2}{\pi+2}$$

$$\frac{2\pi}{4\pi + 8} = \frac{2\pi}{2(2\pi+4)}$$

5 Answer?

Estimate the limit, if it exists: $\lim_{x \to 3} f(x)$ where $f(x)$ represented by the given graph:

(A) 0
(B) –1
(C) 3
(D) 1
(E) The limit does not exist.



6 Answer?

6. Given the function

$$f(x)=\begin{cases}\sin 2x, & x \le \pi \\ 2x+k, & x > \pi\end{cases}$$

what value of k will make this piecewise function continuous?

(A) -2π
(B) $-\pi$
(C) 0
(D) π
(E) 2π

$\lim_{x \to \pi^-} \sin 2x = 2\pi = 0$

$\lim_{x \to \pi^+} 2x+k = 2\pi+k$

$2\pi+k=0$

$k=-2\pi$

7 Answer?

7. Find the limit, if it exists: $\lim_{x\to 0} x\left(e^x + \frac{1}{x}\right)$.

(A) 0
(B) 1
(C) 2
(D) The limit does not exist.
(E) None of these

$\lim_{x\to 0} x\left(\frac{xe^x + 1}{x}\right)$

8 Answer?

8. Identify the vertical asymptotes for $f(x) = \dfrac{x^2 + 3x - 4}{x^2 + x - 2}$.

(A) $x = -2, x = 1$
(B) $x = -2$
(C) $x = 1$
(D) $y = -2, y = 1$
(E) $y = -2$

9 Answer?

9. If $p(x)$ is a continuous function on the closed interval [1, 3], with $p(1) \le K \le p(3)$ and c is in the closed interval [1, 3], then which of the following statements must be true?

(A) $p(c) = \dfrac{p(3) + p(1)}{2}$

(B) $p(c) = \dfrac{p(3) - p(1)}{2}$

(C) There is at least one value c such that $p(c) = K$.

(D) There is only one value c such that $p(c) = K$.

(E) $c = 2$

10 Answer?

10. How many vertical asymptotes exist for the function $f(x) = \dfrac{1}{2\sin^2 x - \sin x - 1}$ in the open interval $0 < x < 2\pi$?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

$$\frac{1}{(2\sin x + 1)(\sin x - 1)}$$

$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

2

$\sin x - 1 = 0$

$\sin x = 1$

1

11 Answer?

x	1	2	3	4
$f(x)$	4	2	3	1
$g(x)$	2	3	1	4

11. Selected values for continuous functions $f(x)$ and $g(x)$ are given in the table above. $\lim_{x \to 3} \frac{f(g(x))}{g(f(x))} =$

(A) $\frac{1}{4}$
(B) $\frac{1}{3}$
(C) 1
(D) 3
(E) 4

$\lim_{x \to 3} \frac{f(g(3))}{f(1)} = 4$

$\lim_{x \to 3} \frac{g(f(3))}{g(3)} = 1$

12 Answer?

12. $\lim_{x \to \infty} \frac{\sin x}{e^x + \cos x} =$

(A) -1

(B) 0

(C) $\frac{1}{e}$

(D) 1

(E) The limit does not exist.

13 Answer?

13. For what value of k is the function $f(x) = \begin{cases} \dfrac{2x^2+5x-3}{x^2-9}, & x \neq -3 \\ k, & x = -3 \end{cases}$ continuous at $x = -3$?

(A) $-\frac{7}{6}$

(B) $-\frac{5}{6}$

(C) 0

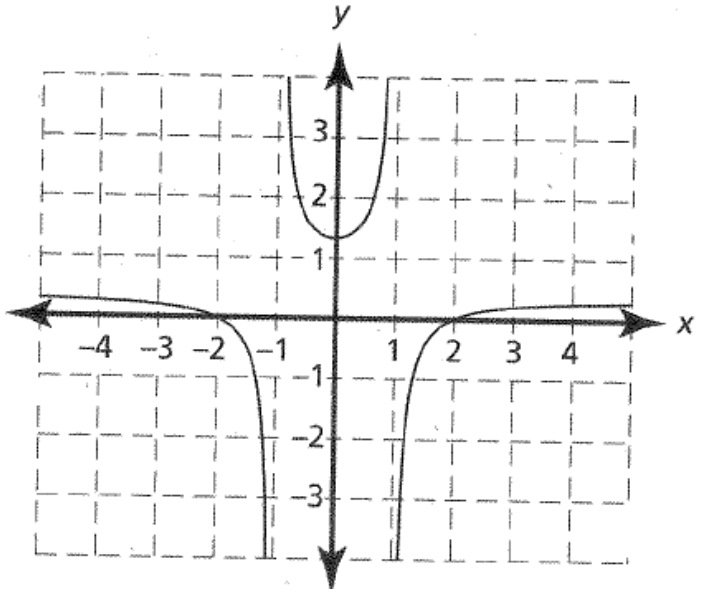
(D) $\frac{5}{6}$

(E) $\frac{7}{6}$

$$\frac{(2x-1)(x+3)}{(x+3)(x-3)}$$

$$-\frac{7}{6} = k$$

14 Answer?



14. The function $g(x)$ is shown in the graph above and is of the form $g(x) = \frac{x^2 + a}{bx^2 - 3}$. Which of the following could be the values of the constants a and b?

(A) $a = -2,\ b = -1$
(B) $a = -2,\ b = -3$
(C) $a = -4,\ b = 3$
(D) $a = -4,\ b = -3$
(E) $a = 4,\ b = 3$

15 Answer?

15. Which of the following statements is true?

(A) $\lim_{x \to 3} \log_3 x = 2$

(B) $\lim_{x \to 0^+} \log_3 x$ does not exist.

(C) $\lim_{x \to -\infty} e^x$ does not exist.

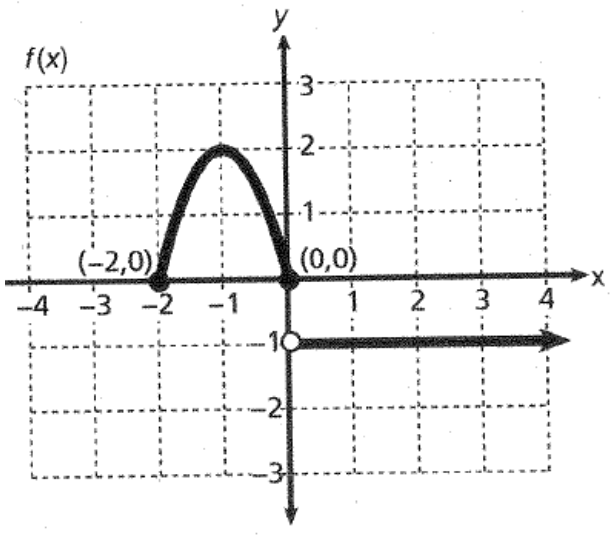
(D) $\lim_{x \to -\frac{\pi}{2}} \csc x = 1$

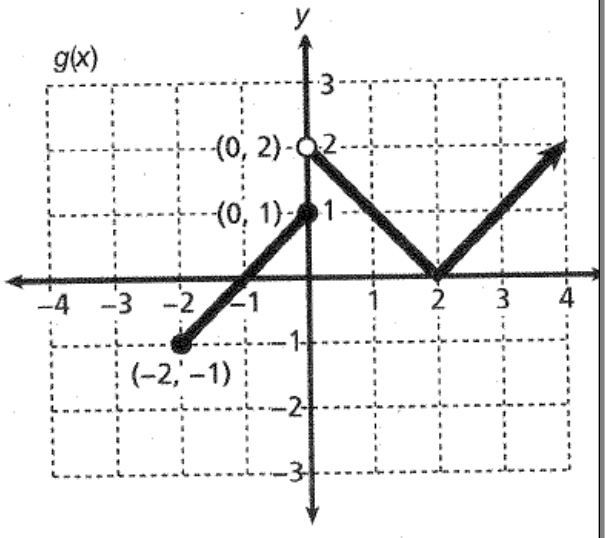
(E) $\lim_{x \to 1} e^{x-1} = 0$

FREE-RESPONSE QUESTION

A calculator may not be used for this question.

1. Use the graphs of $f(x)$ and $g(x)$ given below to answer the following questions:





a. Is $f[g(x)]$ continuous at $x = 0$? Explain why or why not.

b. Is $g[f(x)]$ continuous at $x = 0$? Explain why or why not.

c. What is $\lim_{x \to \infty} f[g(x)]$? Explain your reasoning.

d. If $h(x) = \begin{cases} f(x)+g(x), & -2 \le x \le 0 \\ k+g(x)f(x), & x > 0 \end{cases}$, what is k so that $h(x)$ is continuous at $x = 0$?

Derivative

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Calculators may not be used on this part of the examination.

1 Answer?

1. What does the limit statement $\lim_{x \to 1} \frac{\ln(x+1) - \ln 2}{x-1}$ represent?

(A) 0

(B) $\frac{d}{dx}[\ln(x+1)]$

(C) $f'(1)$, if $f(x) = \ln(x+1)$

(D) 1

(E) The limit does not exist.

2 Answer?

2. Find the derivative of the function $y = \frac{4}{x^3}$.

(A) $-4x^2$

(B) $-\frac{12}{x^2}$

(C) $\frac{12}{x^2}$

(D) $\frac{12}{x^4}$

(E) $-\frac{12}{x^4}$

3 Answer?

3. Find $\frac{dy}{dx}$ if $3xy = 4x + y^2$.

(A) $\frac{4-3y}{2y-3x}$

(B) $\frac{3x-4}{2x}$

(C) $\frac{3y-x}{2}$

(D) $\frac{3y-4}{2y-3x}$

(E) $\frac{4+3y}{2y+3x}$

4 Answer?

4. Find $\frac{dy}{dx}$ for $e^{x+y} = y$.

(A) $\frac{e^{x+y}}{\left(1-e^{x+y}\right)}$

(B) $\frac{e^{x+y}}{\left(1+e^{x+y}\right)}$

(C) $\frac{e^{x+y}}{\left(e^{x+y}-1\right)}$

(D) e^{x+y}

(E) $2e^{x+y}$

5 Answer?

5. If the nth derivative of y is denoted as $y^{(n)}$ and $y = -\sin x$, then $y^{(7)}$ is the same as

(A) y

(B) $\frac{dy}{dx}$

(C) $\frac{d^2y}{dx^2}$

(D) $\frac{d^3y}{dx^3}$

(E) None of these

6 Answer?

6. Find the second derivative of $f(x)$ if $f(x) = (2x+3)^4$.

(A) $4(2x+3)^3$

(B) $8(2x+3)^3$

(C) $12(2x+3)^2$

(D) $24(2x+3)^2$

(E) $48(2x+3)^2$

7 Answer?

A calculator may be used for any of the following multiple-choice questions.

7. Find $\frac{dy}{dx}$ for $y = 4\sin^2(3x)$.

(A) $8\sin(3x)$
(B) $24\sin(3x)$
(C) $8\sin(3x)\cos(3x)$
(D) $12\sin(3x)\cos(3x)$
(E) $24\sin(3x)\cos(3x)$

8 Answer?

If $\ln y = (\ln x)^2 + 2$, find $\frac{dy}{dx}$ in terms of x and y.

(A) $y\left[2\ln(x)+\frac{1}{x}\right]$

(B) $y\left[\left(\frac{2}{x}\right)\ln(x)\right]$

(C) $\left(\frac{2}{x}\right)\ln(x)$

(D) $\frac{2(\ln x)}{x}+2$

(E) $y\left[\frac{2(\ln x)}{x}+2\right]$

9 Answer?

If $f(2) = -3$, $f'(2) = \frac{3}{4}$, and $g(x) = f^{-1}(x)$, what is the equation of the tangent line to $g(x)$ at $x = -3$?

(A) $y - 2 = \frac{-3}{4}(x + 3)$

(B) $y + 2 = \frac{-3}{4}(x - 3)$

(C) $y - 2 = \frac{-4}{3}(x + 3)$

(D) $y + 2 = \frac{4}{3}(x - 3)$

(E) $y - 2 = \frac{4}{3}(x + 3)$

10 Answer?

For what positive value of x does the tangent line to the curve $y = \ln(1 - x)$ intersect the y-axis at the point $(0, 2)$?

(A) 0.382
(B) 0.547
(C) 0.667
(D) 0.722
(E) 0.778

11 Answer?

Calculators may not be used for this part of the examination.

For what values of a and c is the piecewise function

$$f(x)=\begin{cases} ax^2+\sin x, & x \le \pi \\ 2x-c, & x > \pi \end{cases}$$ differentiable?

(A) $a=\dfrac{3\pi}{2}$ and $c=\dfrac{\pi}{2}$

(B) $a=\dfrac{3}{2\pi}$ and $c=\dfrac{7\pi}{2}$

(C) $a=\dfrac{3}{2\pi}$ and $c=-\dfrac{\pi}{2}$

(D) $a=\dfrac{3}{2\pi}$ and $c=\dfrac{\pi}{2}$

(E) $a=\dfrac{3\pi}{2}$ and $c=\dfrac{2}{\pi}$

12 Answer?

If $y = \tan^{-1}(x^2 + 3x)$, then $\frac{dy}{dx} =$

(A) $\frac{1}{1+(x^2+3x)^2}$

(B) $\frac{1}{x^2+3x+1}$

(C) $\frac{2x+3}{1+(x^2+3x)^2}$

(D) $\frac{2x+3}{(x^2+3x)^2}$

(E) $\frac{x^2+3x}{1+(x^2+3x)^2}$

13 Answer?

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	1	−2	4
2	5	3	1	−4
3	2	1	−2	1
4	4	−3	2	−1

Selected function and derivative values for the differentiable functions $f(x)$ and $g(x)$ are given in the table above. If $p(x) = x \cdot f(x) - g(3x-2)$, then $p'(2) =$

(A) 11
(B) 10
(C) 8
(D) 6
(E) 4